Large-Volume String Compactifications, Revisited[⋆]

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ABSTRACT

We reconsider the issue of large-volume compactifications of the heterotic string in light of the recent discoveries about strongly-coupled string theories. Our conclusion remains firmly negative with respect to *classical* compactifications of the ten-dimensional field theory, albeit for a new reason: When the internal sixfold becomes large in heterotic units, the theory acquires an additional threshold at energies much less then the naive Kaluza-Klein scale. It is this additional threshold that imposes the ultimate limit on the compactification scale $M_{\mathrm{KK}} > 4 \cdot 10^7$ GeV *for any compactification*; for most compactifications, the actual limit is much higher. (Generically, $M_{\mathrm{KK}} > \alpha_{\mathrm{GUT}} M_{\mathrm{Planck}}$ in either $SO(32)$ or $E_8 \times E_8'$ heterotic string.)

1. Introduction

From the moment of its re-incarnation as a (candidate) theory of all fundamental interactions, the string theory has always suffered from an embarrassing infinitude of its solutions. To this day, we do not have even a crudest classification of all possible kinds of string vacua. Nevertheless, the oldest known class of such vacua[1] — Kaluza-Klein-like compactifications of a ten-dimensional effective field theory (EFT) — never lost its popularity with string model builders. Originally, the idea was to involve the string theory as little as possible and treat it as simply the ultraviolet cutoff for the EFT, which required the characteristic radius R of the compactified six dimensions to be much larger than the characteristic length scale $\ell_{\text{string}} = \sqrt{\alpha'}$ of the heterotic string, but most modern models of this kind use string-theoretical techniques to analytically continue the model's parameters from $R \gg \ell_{\text{string}}$ to $R \sim \ell_{\text{string}}$[2]. However, our ability to perform such analytic continuation does not answer the old questions of *"How large can the internal manifold be?"* and in particular, *"Can it be large enough to neglect stringy corrections to the EFT at the compactification scale?"*.

In ref. [3], one of the present authors argued that all large-internal-volume compactifications either lead to absurdly small four-dimensional gauge couplings or else require a strongly coupled string theory as well as a ten-dimensional EFT that is strongly-coupled at the string threshold scale. Hence, one could not meaningfully discuss large-volume compactifications in terms of perturbative EFT and perturbative string theory, and since no non-perturbative knowledge of either theory was available at that time, this was the effective end of the discussion. Today however, we do know that the low-energy limit of the ten-dimensional EFT is protected by supersymmetry from any corrections due to high-energy quantum effects, however strong[4]. There is also good evidence that the strong-coupling limit of the heterotic string theory is equivalent to a weakly coupled type I superstring or M-theory (depending on whether the ten-dimensional gauge group is an $SO(32)$ or an $E_8 \times E_8$)[5,6,7]. Thus, it behooves

us to re-visit the old issue of large-volume compactifications and to re-consider the old limit $R \lesssim O(\ell_{\mathrm{string}})$ in light of the new knowledge.

This article is organized as follows: In the next section, we discuss compactifications of the $SO(32)$ heterotic string or its type I superstring dual. We find that for realistic four-dimensional gauge couplings, there are always large stringy corrections at the compactification scale; for the large-internal-volume compactifications, the heterotic string is strongly coupled in spacetime while the dual type I superstring has strong worldsheet couplings. Generically, avoiding unacceptably large stringy quantum corrections to the gauge couplings requires

$$M_{\mathrm{KK}} \gtrsim \alpha_{\mathrm{GUT}} M_{\mathrm{Planck}} \sim 5 \cdot 10^{17} \text{ GeV}. \tag{1.1}$$

However, this limit has loopholes, which allow for essentially unlimited internal volumes of some special compactifications. For any particular compactification, the applicability of the limit (1.1) is determined at the one-loop level of the heterotic string, or dually, at the α'^2 order of the tree-level type I superstring.

Compactifications of the $E_8 \times E_8'$ theory are discussed in section 3. Again, we find that generic compactifications have to satisfy eq. (1.1) in order to prevent the four-dimensional gauge couplings from going haywire, but for some special compactifications the internal volumes are unlimited. From the heterotic point of view, this situation is entirely similar to the $SO(32)$ case, but the dual picture is quite different: The eleventh dimension of the dual M-theory becomes very large in the large volume limit of the other six compact dimensions, and according to E. Witten,[8] the combined compact seven-fold generally does not factorize into a $(S^1/Z_2) \otimes CY^6$. For smooth compactifications, factorization (and hence unlimited volume) require a complete symmetry between the internal gauge fields of the E_8 and the E_8', but the conditions are less stringent for the orbifolds and other singular compactifications.

Section 4 is about non-generic very-large-internal-volume compactifications and their threshold structures. First (section 4.1), we use purely heterotic argu-

ments to show that any such very large compactifications must have some kind of a threshold well below the compactification scale. In the $SO(32)$ case (section 4.2), this sub-Kaluza-Klein threshold turns out to be the type I superstring threshold; consequently, the associated stringy phenomena (Regge trajectories, *etc.*) manifest themselves at much lower energies than the six compact dimensions. As one progresses from lower to higher energies, the physics changes from a $d = 4$ EFT to $d = 4$ string theory to $d = 10$ string theory without ever going through a $d = 10$ EFT regime.

In the $E_8 \times E_8'$ case (section 4.3) there are also two thresholds, albeit of a very different kind: The lower threshold is due to a very large radius ρ of the eleventh dimension of the dual M-theory; at this threshold, the physics changes from a $d = 4$ EFT to a $d = 5$ EFT. At the higher threshold, the other six compact dimensions turn up and the physics changes to a $d = 11$ M-theory regime; again, the $d = 10$ EFT regime does not exist. The intermediate-energy $d = 5$ regime is rather peculiar as the gauge and the matter fields of the supersymmetric Standard Model live on a three-brane at the boundary of the five-dimensional spacetime and only the supergravity and the moduli superfields live in the five-dimensional bulk; there is also 'shadow matter' living on a three-brane at the other end of the fifth dimension. Because of the essentially $d = 4$ nature of the Standard Model in this regime, it is oblivious to the $(d = 4) \rightarrow (d = 5)$ threshold, which thus can be directly probed by gravity or moduli fields only. Indirectly, there are gravitational-strength contributions to gauge bosons' scattering amplitudes due to exachanges of the massive modes of the bulk $d = 5$ fields; such contributions are detectable at the one-loop level of the dual heterotic string, but their actual nature is not apparent at any finite heterotic loop level.

From the phenomenological point of view (section 4.4), one cannot have a string threshold below $O(1 \text{ TeV})$, which implies $M_{\text{KK}} \gtrsim 10^8$ GeV for any $SO(32)$ model. For most string models, there are stronger phenomenological limits associated with baryon stability, neutrino masses, *apparent* trinification of the $SU(3) \times SU(2) \times U(1)$ gauge couplings at 10^{16} GeV, *etc.*, *etc.*, but all of these

limits could in principle be avoided by a sufficiently special model. For the $E_8 \times E_8'$ compactifications, one need not worry about the type I superstring threshold but only about the Kaluza-Klein threshold itself, so the phenomenological limits on M_{KK} are even lower than in the $SO(32)$ case. Surprisingly, the strictest model-independent limit on sizes of the $E_8 \times E_8'$ compactifications comes not from any Standard Model phenomenology but from gravity: Cavendish-type experiments rule out a $(d = 4) \to (d = 5)$ threshold at $\rho \geq 2$ mm,[29] which puts an upper limit on a five-dimensional gravitational coupling, $\kappa_5^2 \lesssim 10^{-23}$ GeV^{-3}, which in turn implies $M_{\mathrm{KK}} \gtrsim 4 \cdot 10^7$ GeV.

The paper concludes with some questions about dynamical supersymmetry breaking in very large compactifications.

2. Compactifications of the $SO(32)$ Theory

We begin with the $SO(32)$ theory in ten dimensions, which appears in the low-energy regime of both the heterotic string and the type I superstring. In terms of the respective string couplings $\lambda_H = \exp(\phi_H)$ and $\lambda_I = \exp(\phi_I)$ and length scales $\ell_H = \sqrt{\alpha_H'}$ and $\ell_I = \sqrt{\alpha_I'}$, the gauge and the gravitational couplings of the ten-dimensional EFT are[⋆]

$$
\begin{aligned}
g_{10}^2 &= \tfrac{1}{2}\lambda_H^2 \ell_H^6 = 4\lambda_I \ell_I^6, \\
\kappa_{10}^2 &= \tfrac{1}{8}\lambda_H^2 \ell_H^8 = (1/16\pi^7)\lambda_I^2 \ell_I^8.
\end{aligned}
\tag{2.1}
$$

Thanks to supersymmetry, these relations are exact and work for both weakly coupled and strongly coupled string theories. In particular, they uphold the heterotic $\leftrightarrow$ type I duality, which relates the couplings and the length scales of the two string theories according to

$$
\hat{\lambda}_I = \frac{1}{\hat{\lambda}_H} \qquad \ell_I = \ell_H \sqrt{\hat{\lambda}_H}
\tag{2.2}
$$

where $\hat{\lambda}_H = \lambda_H/(2\pi)^{7/2}$ and $\hat{\lambda}_I = \lambda_I/16\pi^7$. Notice that whichever of the two

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⋆ In our notations, λ_I^2 corresponds to $g_{\mathrm{open}}^4 = (2\pi)^7 g_{\mathrm{closed}}^2$ of ref. [9]. Also, we normalize the gauge group generators T^a to $tr(Q^a Q^b) = \delta^{ab}$ rather than $2\delta^{ab}$.

dual strings has weaker coupling, it also has a longer length scale. Hence, the energy scale of the threshold between the string theory and the low-energy EFT is located at $1/\ell_H$ when the heterotic string is weakly coupled and the dual type I superstring is strongly coupled, but when the heterotic string is more strongly coupled while the type I superstring is weakly coupled, the threshold is at $1/\ell_I$.

When six out of ten space-time dimensions are compactified to a large internal manifold of volume $V_6 = (2\pi R)^6$,[†] the tree-level couplings of the effective four-dimensional theory are simply

$$\alpha_{\mathrm{GUT}} = \frac{g_4^2}{4\pi} = \frac{g_{10}^2}{4\pi V_6}, \tag{2.3}$$

$$G_N = M_{\mathrm{Planck}}^{-2} = \frac{\kappa_4^2}{8\pi} = \frac{\kappa_{10}^2}{8\pi V_6}.$$

These classical Kaluza-Klein relations are subject to quantum corrections, but let us take them at face value for a moment and consider their implications for the heterotic string and for its type I dual. Substituting eqs. (2.3) into (2.1), we proceed to obtain the string couplings

$$\hat{\lambda}_H = \frac{1}{\hat{\lambda}_I} = \frac{\sqrt{2}}{16} \alpha_{\mathrm{GUT}}^2 (M_{\mathrm{Planck}} R)^3 \tag{2.4}$$

as well as the world-sheet couplings $(\alpha'/R^2) = (\ell/R)^2$ of the two string theories:

$$\left(\frac{\ell_H}{R}\right)^2 = \frac{8}{\alpha_{\mathrm{GUT}}(M_{\mathrm{Planck}} R)^2}, \qquad \left(\frac{\ell_I}{R}\right)^2 = \frac{\alpha_{\mathrm{GUT}}(M_{\mathrm{Planck}} R)}{\sqrt{2}}. \tag{2.5}$$

Furthermore,

$$\hat{\lambda}_H^{2/3} \left(\frac{\ell_H}{R}\right)^2 = \hat{\lambda}_I^{1/3} \left(\frac{\ell_I}{R}\right)^2 = (4\alpha_{\mathrm{GUT}})^{1/3} \sim 1. \tag{2.6}$$

Therefore: *However we choose the Kaluza-Klein scale $M_{\mathrm{KK}} = 1/R$, neither the heterotic string nor the type I superstring can ever be simultaneously weakly*

[†] This notation is not meant to imply that the internal manifold is a torus, it simply serves as a definition of R which we take to be the characteristic Kaluza-Klein length scale.

coupled in space time ($\lambda \ll 1$) and on the world sheet ($\ell \ll R$).[‡] Specifically, for very small manifolds, the heterotic string is weakly coupled in space-time but rather strongly coupled on the world sheet ($\hat{\lambda}_H < 1$ but $\ell_H > R$) while for larger manifolds it is the dual type I superstring that is weakly coupled in space time but strongly coupled on its world sheet ($\hat{\lambda}_I < 1$ but $\ell_I > R$). Consequently, in either case *the string threshold is below the compactification scale.*

In the absence of a Kaluza-Klein-like description, eqs. (2.3) for the four-dimensional couplings in terms of those of a ten-dimensional EFT do not seem to be terribly meaningful, not to mention reliable, but in fact the relations between the $d = 4$ couplings and the string couplings are much more robust. Indeed, let us consider double perturbative expansion of the $d = 4$ gauge and gravitational couplings with respect to both space-time and world-sheet string couplings. In the heterotic case, we have

$$
\begin{aligned}
\frac{1}{\alpha_a} &= \frac{4\,R^6}{\hat{\lambda}_H^2 \ell_H^6} \sum_{n,m} H_{n,m}^a \, \hat{\lambda}_H^{2n} \left(\frac{\ell_H}{R}\right)^{2m}, \\
\frac{1}{G_N} &= \frac{32\,R^6}{\hat{\lambda}_H^2 \ell_H^8} \sum_{n,m} H_{n,m}^g \, \hat{\lambda}_H^{2n} \left(\frac{\ell_H}{R}\right)^{2m},
\end{aligned}
\tag{2.7}
$$

where index a labels simple factors of the $d = 4$ gauge symmetry, $H_{n,m}^{a,g}$ are some model-dependent coefficients; at the tree level of the string $H_{0,m}^g = \delta_{m,0}$ and $H_{0,m}^a = \delta_{m,0} k_a$ where k_a is the corresponding Kac-Moody level; for the $d = 4$ gauge symmetries arising from singular instantons of the $d = 6$ gauge fields, $k_a = 0$. Strictly speaking, the double expansions (2.7) correspond to mutually unrealistic assumptions $\hat{\lambda}_H \ll 1$ *and* $\ell_H \ll R$, but we shall see momentarily that

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[‡] The actual expansion parameter of the perturbative string theory in ten dimensions is $\lambda_H^2/(2\pi)^5 = 4\pi^2 \hat{\lambda}_H^2$ for the heterotic string and $32\lambda/(2\pi)^5 = 16\pi^2 \hat{\lambda}_I$ for the type I superstring. According to eq. (2.6), having $4\pi^2 \hat{\lambda}_H^2 < 1$ at the same time as $\ell_H < R$ would require $\alpha_{\rm GUT} < 1/16\pi^2$, which is incompatible with phenomenological values $\alpha_{\rm GUT} \sim 1/25$. Likewise, having $16\pi^2 \hat{\lambda}_I < 1$ at the same time as $\ell_I < R$ would require $\alpha_{\rm GUT} < 1/64\pi^2$, which is even less compatible with the gauge couplings phenomenology.

the series may be safely extended from that small corner of the parameter space to a much larger area.

Furthermore, the internal manifold does not have to be smooth but may be a large-volume orbifold instead whose orbifold points (or submanifolds) remain singular in the $R \to \infty$ limit. Likewise, the $d = 6$ gauge connection need not be the same as the spin connection and the two connections may even have unrelated singularities (as long as the topological requirements such as $\text{tr}(\mathbf{F} \wedge \mathbf{F}) = \text{tr}(\mathbf{R} \wedge \mathbf{R})$ are satisfied). In general, as long as the singularities make sense in string-theoretical terms and as long as the nature of the singularities remains unchanged in the large-volume limit, the double expansion (2.7) should work.[§]

We presume the string-string duality relations (2.2) to be exact (this has not been proven yet, but the evidence in favor of this assumption is very strong)[10,5,6] and therefore hold in any space-time dimension $d \leq 10$. In terms of the type I superstring's couplings, the heterotic double expansion (2.7) becomes

$$
\begin{aligned}
\frac{1}{\alpha_a} &= \frac{4\,R^6}{\hat{\lambda}_I\,\ell_I^6} \sum_{n,m} H^a_{n,m}\, \hat{\lambda}_I^{m-2n} \left(\frac{\ell_I}{R}\right)^{2m}, \\
\frac{1}{G_N} &= \frac{32\,R^6}{\hat{\lambda}_I^2 \ell_H^8} \sum_{n,m} H^g_{n,m}\, \hat{\lambda}_I^{m-2n} \left(\frac{\ell_I}{R}\right)^{2m}.
\end{aligned}
\tag{2.8}
$$

On the other hand, the type I perturbation theory itself yields a double expansion

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§ Note however that although both *e.g.*, an orbifold and its smooth blow-up would have double expansions (2.7), the two expansions would generally have quite different coefficients.[11] Hence, for our purposes, we should treat the un-blown and the blown-up orbifolds as distinct models whose moduli spaces happen to touch each other. Likewise, we should treat conifolds as distinct from their smooth resolutions as well as smooth deformations, *etc.*. At finite manifold sizes, such models are continuously connected to each other, but in the $R \to \infty$ limit the connections become discontinuous.

of the form

$$\frac{1}{\alpha_a} = \frac{4\,R^6}{\hat{\lambda}_I\,\ell_I^6} \sum_{j,k} I_{j,k}^a\, \hat{\lambda}_I^j \left(\frac{\ell_I}{R}\right)^{2k},$$

$$\frac{1}{G_N} = \frac{32\,R^6}{\hat{\lambda}_I^2\ell_I^8} \sum_{j,k} I_{j,k}^g\, \hat{\lambda}_I^j \left(\frac{\ell_I}{R}\right)^{2k}, \tag{2.9}$$

the Euler number of the type I world sheet being $1 - j$ (for the gauge couplings) or $2 - j$ (for gravity). Exact duality requires exact agreement between the series (2.8) and (2.9), which immediately leads us to the conclusion that

$$I_{j,k}^{a,g} = H_{n,m}^{a,g} \qquad \text{for } k = m = 2n + j. \tag{2.10}$$

Furthermore, since both n and j must be non-negative, the heterotic expansion contains only $H_{n,m}$ with $m \geq 2n$ while the type I expansion has only $I_{j,k}$ with $k \geq j$ and even $k - j$.

This article is about large-radius compactifications, which from the heterotic point of view means $R \ll \ell_H$ while $\hat{\lambda}_H$ may be either small or large. Consequently, for each string loop order n in the double series (2.7), we may truncate the sum over world-sheet loop orders m to the lowest non-trivial order, but because of the heterotic $\leftrightarrow$ type I duality, this order is $m = 2n$ rather than $m = 0$. From the type I point of view, truncation to $m = 2n$ corresponds to truncation to $j = 0$, *i.e.*, to the *tree level of the type I superstring*, which is only natural since according to eqs. (2.4), large R implies small $\hat{\lambda}_I$. Thus, in the large R limit,

$$\frac{1}{\alpha_a} \approx \frac{4\,R^6}{\hat{\lambda}_I\,\ell_I^6} \left[k_a + \sum_{\text{even } k>0} I_{0,k}^a \left(\frac{\ell_I}{R}\right)^{2k} \right],$$

$$\frac{1}{G_N} \approx \frac{32\,R^6}{\hat{\lambda}_I^2\ell_I^8} \left[1 + \sum_{\text{even } k>0} I_{0,k}^g \left(\frac{\ell_I}{R}\right)^{2k} \right] = \frac{32\,R^6}{\hat{\lambda}_I^2\ell_I^8}. \tag{2.11}$$

The last equality here follows from the fact that in the gravitational case, $j = 0$ means a spherical world sheet, on which the degrees of freedom responsible for

the $d = 4$ gravity decouple from those responsible for the internal manifold; consequently, $I_{0,k}^g = 0$ for all $k > 0$. On the other hand, for the gauge couplings $j = 0$ means that the world sheet is a disk, whose boundary (responsible for the type I gauge bosons) may well be entangled with the compactification; consequently, the $I_{0,k>0}^a$ need not vanish.

Phenomenologically, the expansion parameter in series (2.11) is

$$\left(\frac{\ell_I}{R}\right)^4 = \hat{\lambda}_H^2 \left(\frac{\ell_H}{R}\right)^4 = 16\alpha_{\text{GUT}} \left(\frac{R}{\ell_H}\right)^2 = 2\left(\alpha_{\text{GUT}} R M_{\text{Planck}}\right)^2, \quad (2.12)$$

which *increases* rather than decreases with R. Consequently, *there is an upper limit on the size of the internal manifold of a generic compactification* for which string perturbation theory makes any sense,

$$R \leq O\left(\frac{1}{\alpha_{\text{GUT}} M_{\text{Planck}}}\right) \quad \text{or} \quad M_{\text{KK}} \gtrsim \alpha_{\text{GUT}} M_{\text{Planck}} \sim 5 \cdot 10^{17} \, \text{GeV}.$$
$$(2.13)$$

Notice that this limit is somewhat weaker (albeit not much weaker numerically) than the $M_{\text{KK}} \gtrsim \alpha_{\text{GUT}}^{1/6}/\ell_H \sim \alpha_{\text{GUT}}^{2/3} M_{\text{Planck}}$ limit of ref [3] that was based upon naive requirements $g_{10}^2 \lesssim \ell_H^6$ and $\kappa_{10}^2 \lesssim \ell_H^8$, which together amount to $\hat{\lambda}_H \lesssim 1$. On the other hand, the very existence of an upper limit on R and hence on $\hat{\lambda}_H$ contradicts the equally naive argument[12] that in four dimensions, the relevant expansion parameter of the heterotic string is essentially α_{GUT} regardless of $\hat{\lambda}_H$. Instead, the limit (2.13) amounts to a finite, but surprisingly large, limit $\hat{\lambda}_H \lesssim 1/\alpha_{\text{GUT}}$ while the relevant expansion parameter is $\hat{\lambda}_H^2(\ell_H/R)^4$ — a rather obscure combination in heterotic terms. In terms of the dual type I superstring however, the same combination (2.12) has an obvious meaning as the world-sheet expansion parameter. Thus, *the perturbative limit on the internal manifold's size is set not by the heterotic string itself but by its type I dual.*

Unfortunately, the heterotic $\leftrightarrow$ type I string-string duality does not tell us what exactly happens when the manifold's size exceeds the limit (2.13) but only that the perturbation theory breaks down. In order to learn more, let us

henceforth assume that the compactified theory has at least one unbroken super-symmetry in four dimensions. In terms of the four-dimensional EQFT (Effective QFT), the size of the internal manifold manifests itself through the Kähler moduli T_i; perturbatively, $\mathrm{Im}\, T_i \sim R^2/\ell_H^2$. The *Wilsonian* gauge couplings of an $N = 1$ supersymmetric EQFT must be holomorphic (or rather harmonic) functions of the chiral superfields of the theory.[13,14] Combining this requirement with the invariance under discrete Peccei-Quinn symmetries $T_i \to T_i + 1$ and $S \to S + 1$, one finds that in the large R limit, the Wilsonian gauge couplings must behave according to[11]

$$\frac{1}{\alpha_a^W} = k_a \,\mathrm{Im}\, S + \sum_i C_{a,i}\,\mathrm{Im}\, T_i + \mathrm{const} + O\big(e^{-2\pi\,\mathrm{Im}\, T_i}, e^{-2\pi\,\mathrm{Im}\, S}\big), \quad (2.14)$$

where $C_{a,i}$ are $O(1)$ rational coefficients determined at the one-heterotic-string-loop level of any particular model. Indeed, in heterotic terms, the $k_a \,\mathrm{Im}\, S$ terms appear at the tree level, the $C_{a,i}\,\mathrm{Im}\, T_i$ and other S-independent terms appear at the one-loop level, nothing whatsoever appears at the higher-loop orders and the non-perturbative terms are exponentially small.[⋆]

At the tree level, the chiral dilaton/axion superfield is well defined and its dilaton component $\mathrm{Im}\, S$ may be identified with $1/\alpha_{\mathrm{GUT}}$. At the quantum level however, one is generally free to shift S by a linear combination (with integer coefficients) of the moduli T_i plus a power series in $e^{2\pi i T_i}$. Such a shift amounts to a re-definition of the 'unified' gauge coupling as

$$\frac{1}{\alpha_{\mathrm{GUT}}^W} = \mathrm{Im}\, S + \sum_i \nu_i\,\mathrm{Im}\, T_i + \mathrm{const} + O\big(e^{-2\pi\,\mathrm{Im}\, T_i}, e^{-2\pi\,\mathrm{Im}\, S}\big); \quad (2.15)$$

consequently, the large R limits of the Wilsonian gauge couplings α_a^W can be

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[⋆] To be precise, eq. (2.14) applies to all $d = 4$ gauge couplings, including those of the non-perturbative gauge fields. For such couplings, $k_a = 0$ and the S-independent terms in eq. (2.14) arise non-perturbatively rather than at the one-loop level.

summarized as

$$\frac{1}{\alpha_a^W} = \frac{k_a}{\alpha_{\text{GUT}}^W} + C_a \frac{R^2}{\ell_H^2} + O(1) \tag{2.16}$$

where the coefficients C_a depend on the shape of the internal manifold (via ratios of T_i to R^2/ℓ_H^2) and on the matrix $\tilde{C}_{a,i} = C_{a,i} - k_a \nu_i$.

The Wilsonian couplings such as (2.14) are parameters of the defining Lagrangian of a low-energy EQFT from which massive string modes are integrated out but the light fields remain fully quantized and their quantum effects are yet to be taken into account. On the other hand, the string-theoretical low-energy couplings such as (2.11) are physical couplings that account for all quantum effects, both high-energy and low-energy. Hence, a proper comparison between two kinds of couplings involves adding the purely field-theoretical quantum corrections to the Wilsonian couplings. Without going through the sordid details of such corrections,[15,14,11] let us simply describe their behavior in the large R limit: At the one-loop level of the $d = 4$ EQFT,

$$\frac{1}{\alpha_a(M_{\text{string}})} = \frac{1}{\alpha_a^W} + O\left(\log \frac{R^2}{\ell_H^2}\right) \tag{2.17}$$

while the higher-loop corrections are suppressed by powers of α_{GUT} times a largish logarithm.[†] Notice the logarithmic growth of field-theoretical corrections (2.17) with the radius R is much slower than the generally quadratic growth of the Wilsonian gauge couplings (2.16). Hence, all we really need to know in order to understand the large radius behavior of a physical gauge coupling α_a is the sign of the coefficient C_a in eq. (2.16):

- If $C_a < 0$, then the coupling α_a increases with the radius; for sufficiently

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† We assume that none of the physical Yukawa couplings of the EQFT grows like a positive power of R/ℓ_H. If there were such a rapidly growing Yukawa coupling, the model could not be continued to large R regardless of what happened to the gauge couplings.

large

$$R_{\text{max}}^2 \approx \frac{k_a}{-C_a} \frac{\ell_H^2}{\alpha_{\text{GUT}}}, \tag{2.18}$$

α_a becomes infinite and the theory has some kind of a phase transition near the limit (2.13).

$\diamond$ On the other hand, if $C_a > 0$, then the coupling α_a decreases with R and for the radii much large than the limit (2.13), we have $\alpha_a \ll \alpha_{\text{GUT}}$ thus robbing α_{GUT} of its physical meaning as an overall measure of all $d = 4$ gauge couplings. More to the point, such exceedingly weak gauge couplings would be inconsistent with the known phenomenology. (Unless they belong to hitherto undiscovered hidden sectors, but then such hidden sectors would be quite useless for dynamical supersymmetry breaking.)

Since the exact definition of the 'unified' gauge coupling α_{GUT} for any particular model is somewhat arbitrary, a convenient choice of coefficients ν_i in eq. (2.15) would let us set $C_a = 0$ for any particular gauge coupling α_a; alternatively, we may make all the C_a non-negative, or non-positive. Generically, however, no choice of the ν_i would make all the C_a vanish at the same time, so however we define α_{GUT}, if we keep it fixed while R increases beyond the limit (2.13), at least some of the α_a would become either too strong or too weak. In other words, *generically, eq. (2.13) gives a physical limit on the internal manifold's size beyond which the theory cannot be continued.* However, for some models we may be able to make all the C_a vanish; such *special models may be continued to arbitrarily large radii R* (or at least to exponentially large $R \sim \ell_H \exp(1/\alpha_{\text{GUT}})$).

Let us now consider this result from the dual type I point of view. Since ℓ_I increases with R faster than R itself while the type I superstring coupling $\hat{\lambda}_I$ becomes small, the large R behavior of the gauge couplings is dominated by the world-sheet quantum effects at the tree level of the type I superstring. Furthermore, comparing the expansion (2.11) with the four-dimensional result

(2.16) (plus the fact that the EQFT loop corrections are sub-leading relative to the (2.16) in the large R limit), we immediately arrive at.

$$I_{0,0}^a = k_a, \qquad I_{0,2}^a = \frac{C_a}{16}, \qquad \text{all other } I_{0,k}^a = 0. \qquad (2.19)$$

In other words, *at the tree level of the type I superstring compactified to four dimensions, the only orders in the α_I'/R^2 expansion contributing to the gauge couplings are the zeroth and the second.* Furthermore, *a model may be continued to large radii R if and only if the second order (in α_I'/R^2) contributions to all the gauge couplings happen to vanish.*

Unfortunately, as of this writing, we can only state these results as our predictions as to what an actual type I calculation should yield, but we do not have any "experimental" verifications of these predictions. Eventually, we hope to understand the internal $d = 6$ gauge fields from the type I point of view well enough to calculate their effect on the $d = 4$ gauge fields in a generic compactification, but at the moment we only understand the somewhat trivial case: An $SO(N)$ subgroup of the $SO(32)$ arising from N out of 32 Chan-Patton factors that simply do not get involved in the compactification in any way. Obviously, at the tree (disk) level of the type I superstring such a subgroup is simply unaffected by any details of the compactification and thus has $\alpha = \alpha_{\text{GUT}}$ regardless of the internal manifold's size or shape. Thus, such a subgroup not only agrees with eq. (2.19), but would also impose no limit on the internal manifold's size. It would be very interesting to find other kinds of $d = 4$ gauge symmetries that behave in this way.

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3. Compactifications of the $E_8 \times E_8'$ Theory

Thus far, we have discussed compactifications of the ten-dimensional $SO(32)$ theory; let us now turn our attention to the $E_8 \times E_8'$ case. The $d = 10$ effective field theory with this gauge group emerges in the low-energy regime of the heterotic string and also of the eleven-dimensional M-theory compactified on a semi-circle. According to Horava and Witten[7,16], in the latter case the ten-dimensional couplings are

$$
\begin{aligned}
g_{10}^2 &= 2\pi(4\pi\kappa_{11}^2)^{2/3}, \\
\kappa_{10}^2 &= \frac{\kappa_{11}^2}{2\pi\rho},
\end{aligned}
\tag{3.1}
$$

where κ_{11} is the gravitational coupling of the $d = 11$ M-theory and ρ is the radius of the semi-circle on which the eleventh dimension is compactified. Comparing these couplings to those of the dual heterotic string (eq. (2.1)), we find

$$
\alpha_H' \equiv \ell_H^2 = \frac{2\ell_{11}^3}{\rho}, \qquad \hat{\lambda}_H = \frac{1}{2}\left(\frac{\rho}{\ell_{11}}\right)^{3/2},
\tag{3.2}
$$

where we have conveniently if arbitrarily identified the eleven-dimensional length scale ℓ_{11} according to

$$
4\pi\kappa_{11}^2 = (2\pi\ell_{11})^9.
\tag{3.3}
$$

Let us now compactify the ten-dimensional $E_8 \times E_8'$ theory on a large $d = 6$ internal manifold. As in the $SO(32)$ case, we do not require this manifold to be smooth or the gauge connection to equal the spin connection, but only that the singularities do not change their nature in the large R limit (*i.e.*, the orbifolds remain orbifolds and do not get blown-up, *etc.*). In the Kaluza-Klein limit, when R is larger than any ten-dimensional threshold scale, the four-dimensional couplings are given by eqs. (2.3). Combining those equations with eqs. (3.1) and

solving for the $d = 11$ length scale and the semi-circle radius, we obtain

$$\ell_{11} \; = \; \left(2\alpha_{\text{GUT}}\right)^{1/6} R, \qquad \rho \; = \; \left(\tfrac{1}{2}\alpha_{\text{GUT}}\right)^{3/2} M_{\text{Planck}}^2 R^3. \qquad (3.4)$$

Curiously, for any size of the internal manifold, the eleven-dimensional length scale ℓ_{11} is always just a bit shorter than the compactification scale R; numerically, for phenomenological values $\alpha_{\text{GUT}} \sim 1/25$, we have $\ell_{11} \approx 0.65R$. Or, from the super-membrane point of view (assuming that the M-theory is some kind of a supermembrane theory), the world-brane coupling

$$\left(\frac{\ell_{11}}{R}\right)^2 \; = \; \sqrt[3]{2\alpha_{\text{GUT}}} \qquad (3.5)$$

is smallish but not particularly small numerically. Hence, a semi-classical Kaluza-Klein-like treatment of the six compact dimension of the M-theory should be qualitatively valid but perhaps not too accurate quantitatively — except for the quantities protected from the world-brane quantum corrections by an unbroken supersymmetry.

Next consider the semi-circle radius ρ: According to eq. (3.4), it grows like R^3 while the other six compact dimensions grow like R. Thus, for $R \gtrsim (7 \cdot 10^{17}\,\text{GeV})^{-1}$, ρ becomes larger than R and *the lowest-energy threshold is ρ rather than R!* Consequently, in this regime, we would have four-dimensional physics at low energies below $1/\rho$, *five-dimensional physics at intermediate energies* between $1/\rho$ and $1/R$ and *eleven-dimensional physics at high energies* above $1/R$. At no energies however would we find the ten-dimensional physics, semi-classical or otherwise!

Before we proceed any further, we should consider the validity of eqs. (3.4) for large R compactifications. Although the heterotic string $\leftrightarrow$ M-theory duality relations (3.2) are presumably exact and therefore remain valid after any compactification to $d < 10$, in the absence of a $d = 10$ effective field theory, the Kaluza-Klein eqs. (2.3) do not make much sense. However, as in the $SO(32)$ case

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discussed in the previous section, we may use the $d = 4$ supersymmetry and the discrete Peccei-Quinn symmetries of the four-dimensional EQFT to derive the relations between the $d = 4$ couplings and the string / M-theory couplings in a way that does not depend on any $d = 10$ effective theory. Indeed, our previous analysis of the $SO(32)$ heterotic string may be repeated verbatim for the present $E_8 \times E_8'$ case to yield once again

$$\frac{1}{\alpha_a^W} = \frac{k_a}{\alpha_{\text{GUT}}^W} + C_a \frac{R^2}{\ell_H^2} + O(1), \tag{2.16}$$

from which we again conclude that *generically, eq. (2.13) gives a physical limit on the internal manifold's size beyond which the theory cannot be continued, but special models may be continued to exponentially large radia R.*

From the M-theory point of view however, the reason generic $E_8 \times E_8'$ models break down at large R is very different from the $SO(32)$ case: Unlike the world-sheet coupling $(\ell_I/R)^2$ of the type I superstring that becomes large for large R, the world-brane coupling (3.5) of the supermembrane remains moderately small. Furthermore, the world-brane coupling $(\ell_{11}/\rho)^2$ due to compactness of the eleventh dimension becomes very small in the large R limit, so it could not possibly cause any breakdown. It is the largeness rather than smallness of ρ that causes a breakdown of a very different kind: The seven compact dimensions no longer form a direct product of a semicircle and a Calabi-Yau sixfold but rather a sevenfold (with boundaries) whose metric depends on all seven coordinates in a non-trivial way; likewise, the 3-form field of the M-theory also depends on all seven coordinates. This breakdown of factorization was discovered by E. Witten and we have little to add to his exposition in ref. [8]. We would like however to comment on his formula for the gauge couplings for the unbroken subgroups of the E_8 or the E_8', which in present notations becomes

$$\frac{1}{k_a \alpha_a} = \frac{2R^6}{\ell_{11}^6} + \frac{\rho}{64\pi^4 \, \ell_{11}^3} \int\limits_{CY} \omega_K \wedge \left(\text{tr}(\mathbf{F} \wedge \mathbf{F}) - \tfrac{1}{2} \text{tr}(\mathbf{R} \wedge \mathbf{R}) \right) + \cdots \tag{3.6}$$

where $\mathbf{F}$ is the $d = 6$ gauge field strength of whichever E_8 happens to contain

the subgroup in question, $\mathbf{R}$ is the $d = 6$ curvature form, ω_K is the Kähler form of the Calabi-Yau sixfold and the '$\cdots$' stand for the sub-leading terms in the large-R-large-ρ limit. The first term on the right hand side here is clearly $1/\alpha_{\text{GUT}}$ while the second term in the dual heterotic units becomes

$$\frac{1}{32\pi^4\alpha'_H} \int_{CY} \omega_K \wedge \left(\text{tr}(\mathbf{F} \wedge \mathbf{F}) - \tfrac{1}{2} \text{tr}(\mathbf{R} \wedge \mathbf{R}) \right) \propto \frac{R^2}{\alpha'_H}, \qquad (3.7)$$

in full agreement with eq. (2.16) and the fact that the EQFT quantum corrections are sub-leading in the large R limit. Furthermore, without actually performing any one-string-loop calculations in the heterotic theory, we may extract the values of the coefficients $C_{a,i}$ from the dual M-theory by simply decomposing the expressions (3.7) for the two E_8 factors in terms of the independent Kähler moduli $\text{Im}\, T_i$ and corresponding (1,1) forms ω_i:

$$\begin{aligned}
\omega_K &= \sum_i \omega_i (2\alpha'_H \,\text{Im}\, T_i), & \frac{C_a R^2}{\alpha'_H} &= \sum_i (C_{a,i} - k_a \nu_i) \,\text{Im}\, T_i, \\
\tilde{C}_{a,i} &\equiv C_{a,i} - k_a \nu_i = \frac{k_a}{(2\pi)^4} \int_{CY} \omega_i \wedge \left(\text{tr}(\mathbf{F} \wedge \mathbf{F}) - \tfrac{1}{2} \text{tr}(\mathbf{R} \wedge \mathbf{R}) \right).
\end{aligned} \qquad (3.8)$$

In the special case of a (2,2) compactification where $\text{tr}(\mathbf{F} \wedge \mathbf{F})_1 = \text{tr}(R \wedge R)$ and the first E_8 is broken down to the E_6 while $\mathbf{F}_2 = 0$ and the E'_8 remains unbroken, eqs. (3.8) reproduce the "topological" string-threshold correction of Bershadsky, Cecotti, Ooguri and Vafa[17]

$$\frac{1}{\alpha_6} - \frac{1}{\alpha_8} = 12 F_1.$$

Generalization of this formula to a more general case where the manifold is large and smooth and the $d = 6$ gauge fields are non-singular and restricted to simple subgroups of the $E_8 \times E'_8$ but are not otherwise restricted (except for the topological constraints) is quite straightforward and the result is again in full agreement with eq. (3.8); this serves as yet another confirmation of the duality

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between the M-theory and the heterotic string. It would be interesting however to extend this confirmation to the singular compactifications as well.

We conclude this section by noticing that the M-theory makes for a rather simple criterion for the special compactifications whose sizes are not limited by eq. (2.13): The $d = 6$ fields belonging to the two E_8 factors (from the nine-branes at each end of the eleventh dimension) should be cohomologically equal to each other. That is,

$$
\int_\Sigma \mathrm{tr}(\mathbf{F} \wedge \mathbf{F})_1 \;=\; \int_\Sigma \mathrm{tr}(\mathbf{F} \wedge \mathbf{F})_2 \;=\; \frac{1}{2} \int_\Sigma \mathrm{tr}(\mathbf{R} \wedge \mathbf{R}) \tag{3.9}
$$

for every *large* closed 4-cycle Σ of the Calabi-Yau sixfold. By 'large' we mean that the corresponding Kähler moduli $\mathrm{Im}\, T_i$ grow like $O(R^2/\ell_H^2)$ while the cycle's 4-volume grows like $O(R^4)$; this excludes from consideration the cycles surrounding orbifold points and other singularities that do not get blown up in the large R limit. Therefore, while the *smooth* large-radius compactifications of the $E_8 \times E_8'$ theory require $\mathbf{F}_1 = \mathbf{F}_2$, which implies complete symmetry between the two E_8 gauge groups and hence 'shadow matter', exactly like ours, at the other end of the eleventh dimension, the large but singular compactifications may have $\mathbf{F}_1 \neq \mathbf{F}_2$ and hence shadow matter that is quite different from the Standard Model.

Note however that the 'left-right symmetric' orbifolds or any other $(2,2)$ compactifications in which the E_8' is completely unbroken are never allowed to grow very large since they cannot satisfy eqs. (3.9) for any 4-cycle (and there are always 4-cycles that grow like R^4, *e.g.*, toroidal cycles of an orbifold).

4. Very Large Internal Sixfolds

In the previous sections, we saw that while generic compactifications of the heterotic string are limited to sizes $R \lesssim 1/\alpha_{\text{GUT}} M_{\text{Planck}}$, there are also some special models in which $\alpha_{\text{GUT}} \sim 1/25$ can peacefully coexist with arbitrarily large radia R. In all such models however, there is a threshold at energies much lower than the Kaluza-Klein scale $M_{\text{KK}} = 1/R$: In the $SO(32)$ case, there is a type I superstring threshold at

$$M_I = \frac{1}{\ell_I} = \frac{2^{1/4} M_{\text{KK}}^{3/2}}{(\alpha_{\text{GUT}} M_{\text{Planck}})^{1/2}}, \tag{4.1}$$

while in the $E_8 \times E_8$ there is a $(d=4) \to (d=5)$ decompactification at

$$M_5 = \frac{1}{\rho} = \frac{2^{3/2} M_{\text{KK}}^3}{\alpha_{\text{GUT}}^{3/2} M_{\text{Planck}}^2}. \tag{4.2}$$

This section is about the effect of such thresholds on the ordinary four-dimensional physics and the consequent phenomenological limits on the internal sixfold's size.

4.1 HETEROTIC EVIDENCE

The gauge couplings α_a we have discussed in the previous sections correspond to the most relevant $\text{tr} \, F_{\mu\nu}^2$ terms in the effective Lagrangian for the gauge bosons. The higher derivative/order terms such as $\text{tr} \, F_{\mu\nu}^4$ are irrelevant to the low-energy regime of the effective $d = 4$ theory, but they are very relevant to its high-energy limitations: When at sufficiently high energies the higher derivative/order terms have as much effect on various amplitudes as the lowest derivative/order terms, the low-energy effective theory reaches its limit and there must be some kind of a threshold. Therefore, as our first estimate of the lowest threshold scale in large-size compactifications of the heterotic string, we shall now proceed to calculate the $\text{tr} \, F_{\mu\nu}^4$ terms for the four-dimensional gauge fields.

At the tree level of the heterotic string, there are no $\operatorname{tr} F_{\mu\nu}^4$ terms, but they do appear at the one-loop and higher orders. The supersymmetry severely restricts quantum corrections to the coefficients of the lowest-derivative $\operatorname{tr} F_{\mu\nu}^2$ terms, but the $\operatorname{tr} F_{\mu\nu}^4$ terms are not subject to such restrictions. Indeed, even the ten-dimensional supersymmetry which completely forbids any quantum corrections to the ordinary gauge couplings allows for the quadratically divergent one-loop renormalization of the four-derivative couplings in $d = 10$ QFT. In the heterotic string theory, the ultraviolet divergence is cut off, which leads to a finite $O(1/\alpha_H')$ four-derivative coupling. The actual one-string-loop calculation was done by Ellis, Jetzer and Mizrachi,[18] who found

$$
\mathcal{L}_{1\,\text{loop}}^{d=10} \supset -\frac{\tau^{1234}}{6144\pi^5\,\alpha_H'}
\begin{cases}
\operatorname{tr}(F_1 F_2 F_3 F_4) & \text{for } SO(32), \\[2ex]
\frac{1}{4}\begin{bmatrix} \operatorname{tr}(F_1 F_2)\operatorname{tr}(F_3 F_4) \\ -\operatorname{tr}(F_1 F_2)\operatorname{tr}(F_3' F_4') \\ +\operatorname{tr}(F_1' F_2')\operatorname{tr}(F_3' F_4') \end{bmatrix} & \text{for } E_8 \times E_8',
\end{cases}
\tag{4.3}
$$

where $1, 2, 3, 4$ are short-hand notations for anti-symmetric pairs of space-time indices $\mu_1\nu_1$ through $\mu_4\nu_4$ and

$$
\tau^{1234} = \sum_{\substack{\text{permutations} \\ \text{of } 1,2,3,4}} \left[g^{\nu_1\mu_2} g^{\nu_2\mu_3} g^{\nu_3\mu_4} g^{\nu_4\mu_1} - \tfrac{1}{4} g^{\mu_1\mu_2} g^{\nu_1\nu_2} g^{\mu_3\mu_4} g^{\nu_3\nu_4} \right]
\tag{4.4}
$$

is an $SO(9,1)$ invariant tensor totally symmetric in four such pairs; in the $E_8 \times E_8'$ case, 'tr' denotes $\frac{1}{30}$ of the trace over the adjoint representation of the appropriate E_8. Curiously, when the gauge fields $F_{\mu\nu}$ are restricted to a Cartesian ($k = 1$) $SU(2)$ subgroup of either $SO(32)$ or $E_8 \times E_8'$, both heterotic string theories yield identical $F_{\mu\nu}^4$ interactions, although this does not apply to the more general gauge fields whose gauge indices may be contracted in different ways.

For our purposes, we need the $F_{\mu\nu}^4$ couplings of the four-dimensional gauge bosons of the heterotic string compactified on a large sixfold. To calculate such coupling, we may follow exactly the same procedure as Ellis, Jetzer and Mizrachi,

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the only difference being in the partition functions of the various sectors of the heterotic string. In the large R limit, the four-dimensional partition functions are related to their ten-dimensional counterparts by the overall factor $V_6 = (2\pi R)^6$ times a sector-dependent correction

$$1 \; + \; O\left(\frac{\alpha_H' \operatorname{Im} \tau}{R^2}\right) \tag{4.5}$$

where τ is the modular parameter of the one-loop world sheet. Such sector-dependent correction factors may spoil supersymmetric cancellations between different sectors and thus are very important for quantities that would be forbidden by $d = 10$ supersymmetry but are allowed by $N < 4$ supersymmetry in $d = 4$. Likewise, the correction factors (4.5) would be important for the low-energy loops corresponding to $\alpha' \operatorname{Im} \tau \gtrsim R^2$. Fortunately, neither condition applies to the four-derivative gauge couplings, which arise from the high-energy loops (corresponding to $\operatorname{Im} \tau = O(1)$) and are not subject to supersymmetric cancellations. Consequently, in the large R limit the sector-dependent factors (4.5) become unimportant and the four-dimensional calculation proceeds exactly as in ten dimensions, yielding precisely (4.3) times an overall six-volume factor $(2\pi R)^6$.

The above analysis leads to $O(R^6/\alpha_H')$ coefficients of the dimension eight operators $F_{\mu\nu}^4$ in the four-dimensional effective Lagrangian. Naively, such operators become important at energies $E \gtrsim \ell_H^{1/2}/R^{3/2}$, which immediately indicates a threshold well below the Kaluza-Klein scale $M_{\mathrm{KK}} = 1/R$. The reason this estimate is naive is that it does not take into account the non-canonical normalization of the gauge fields; a more accurate estimate would require comparing scattering amplitudes due to the $F_{\mu\nu}^4$ operators to the amplitudes due to the non-abelian part of the usual $F_{\mu\nu}^2$ Lagrangian. For example, consider a four-point scattering amplitude for the gauge bosons belonging to the same $SU(2)$ subgroup of the four-dimensional gauge symmetry, for which the relevant part

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of the low-energy effective Lagrangian can be summarized as

$$\mathcal{L}_{SU(2)}^{d=4} \;=\; \frac{-1}{8\pi\alpha}\ \mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) \;-\; \frac{\pi R^6}{96\alpha_H'}\ \tau^{1234}\ \mathrm{tr}(F_1 F_2)\ \mathrm{tr}(F_3 F_4). \qquad (4.6)$$

To be precise, this effective Lagrangian already includes both high-energy and low-energy loop corrections, so the scattering amplitudes follow from the tree-level Feynman graphs only. With a bit of algebra, one can show that for the four-gauge-boson amplitudes,

$$\frac{\mathcal{A}[F_{\mu\nu}^4]}{\mathcal{A}[F_{\mu\nu}^2]} \;=\; -\frac{2\pi^2 \alpha R^6}{3\alpha_H'}\ st \qquad (4.7)$$

where s and t are Mandelstam's kinematic variables. This amplitude ratio increases with energy as E^4 (for a fixed scattering angle); at

$$E_t \;\sim\; \left(\frac{\alpha_H'}{\alpha R^6}\right)^{1/4} \;\sim\; \frac{M_{\mathrm{KK}}^{3/2}}{(\alpha_{\mathrm{GUT}} M_{\mathrm{Planck}})^{1/2}} \qquad (4.8)$$

the effect of the higher-derivative operators on scattering can no longer be neglected and the low-energy effective theory reaches a threshold.

Note that eq. (4.7) holds for both $SO(32)$ and $E_8 \times E_8'$ heterotic strings. In the heterotic case, the apparent threshold scale (4.8) is similar to the dual type I superstring scale (4.1), and in the next section we shall see that the threshold indicated by the $F_{\mu\nu}^4$ operators is indeed the type I superstring threshold. The appearance of the same threshold scale in the $E_8 \times E_8'$ case is much more mysterious; we shall return to this issue in section 4.3.

4.2 VERY LARGE COMPACTIFICATIONS OF THE TYPE I SUPERSTRING

In order to identify the apparent threshold (4.8) of the $SO(32)$ heterotic string theory as the string threshold (4.1) of the dual type I superstring we need to answer two basic questions:

1. Do the $F_{\mu\nu}^4$ couplings of the heterotic string and the type I superstring agree with each other?

2. Is the heterotic estimate based on the one-loop $F_{\mu\nu}^4$ couplings reliable? Specifically, are there higher-loop contributions to such couplings that are stronger than $(4.3) \times (2\pi R)^6$? (Note for the large-size compactifications $\hat{\lambda}_H \gg 1$.) Also how strong are the six- and higher derivative couplings $F_{\mu\nu}^6$, *etc.*? — If they are strong enough, there should be a threshold at energies below (4.8).

Let us begin to answer these questions by first considering what happens in ten uncompactified dimensions. According to Tseytlin,[20] the heterotic $\leftrightarrow$ type I duality indeed works for the $F_{\mu\nu}^4$ couplings in $d = 10$; furthermore, in the heterotic theory such couplings arise solely at the one-loop level while in the type I theory they arise at the tree level only[19,21]. Consequently, when the six dimensions are compactified in a manner that does not affect N out of 32 Chan-Patton degrees of freedom living on the open boundaries of the type I worldsheets, the corresponding $SO(N)$ subgroup of the $SO(32)$ would be totally unaffected by the compactification at the tree (disc) level of the type I superstring. Instead, all tree-level $F_{\mu\nu}^2$, $F_{\mu\nu}^4$, $F_{\mu\nu}^6$, *etc.*, couplings for such a subgroup in $d = 4$ would be precisely equal to their $d = 10$ counterparts times $(2\pi R)^6$. As we already mentioned, the same is true for the heterotic one-loop $F_{\mu\nu}^4$ couplings in the large R limit; consequently, Tseytlin's duality between the ten-dimensional heterotic and type I couplings extends straightforwardly to the large-size compactifications.

Clearly, the above argument is limited to the simplest kind of gauge theories of the compactified type I superstring. These, alas, are the only gauge theories

that we presently know how to extend to the large R limit where $\ell_I \gg R$. All other kinds of $d = 4$ gauge theories are understood in type I terms only for $\ell_I \ll R$ and generally are expected to have phase transitions for $\ell_I \sim R$. However, it is perfectly possible that some special theories of this kind are consistent with very large radius compactifications and it would be very interesting to find out what happens to the higher-derivative couplings of such theories at large R.

Let us now presume exact heterotic $\leftrightarrow$ type I duality and use it to answer our second question concerning the reliability of the apparent threshold scale based solely on the heterotic one-loop $F_{\mu\nu}^4$ couplings. Following the procedure we used in section 2, we write down double perturbative expansions for all $F_{\mu\nu}^A$ couplings ($A = 2, 4, \ldots$) and demand that the two dual string theories agree with each other. Suppressing all gauge and space-time indices, the coefficient $\mathcal{F}_A$ of an $F_{\mu\nu}^A$ coupling expands to

$$
\begin{aligned}
\mathcal{F}_A &= R^6 \ell_H^{2A-10} \sum_{m,n} \mathcal{H}_{m,n}^A \, \hat{\lambda}_H^{2n-2} (\ell_H/R)^{2m} \\
&= R^6 \ell_I^{2A-10} \sum_{j,k} \mathcal{I}_{j,k}^A \, \hat{\lambda}_I^{j-1} (\ell_I/R)^{2k}
\end{aligned}
\tag{4.9}
$$

where the first sum is the heterotic double expansion, the second is the type I double expansion and the overall factors $R^6 \ell^{2A-10}$ follow from the canonical dimension of the operator $F_{\mu\nu}^A$ and from having four non-compact and six compact spacetime dimensions. Making use of the duality relations (2.2) between the string couplings and length scales and demanding exact agreement between the two double expansions, we arrive at

$$
\mathcal{H}_{m,n}^A = \mathcal{I}_{j,k}^A \qquad \text{for} \quad k = m = 2n + j + 2 - A
\tag{4.10}
$$

(*cf.* eq. (2.10) for $A = 2$). As in section 2, we are concerned with $R \gg \ell_H$ and hence with smallest possible m for each heterotic loop level n. Again, such smallest possible m corresponds to $j = 0$, so in terms of the dual type I

superstring, only the tree-level contributions are important in the large R limit. Generically, at this stage we are left with a power series in a large parameter (2.12) and no analytic way to sum the series. However, in the special case where the $d = 4$ gauge symmetry decouples from the compactification at the tree level of the type I superstring, $k > 0$ are not allowed for $j = 0$ and hence

$$\mathcal{F}_A \sim R^6 \ell_I^{2A-10}. \tag{4.11}$$

From the heterotic point of view, $k = j = 0$ implies $m = 0$ and $n = (A-2)/2$. Thus, the usual $F_{\mu\nu}^2$ gauge couplings arise at the tree level of the heterotic string and are largely unaffected by the loop corrections (for the special gauge couplings only!). Similarly, the $F_{\mu\nu}^4$ couplings arise at the one-loop level and are largely unaffected by the higher loops, which retroactively justifies the analysis of the previous section. Likewise, the six-derivative couplings $F_{\mu\nu}^6$ arise at the two-loop level and are largely unaffected by the still higher loop orders, *etc.*

Finally, when all heterotic loop orders and all higher-derivative gauge couplings are taken into account, their combined contributions to the gauge boson scattering amplitudes are nothing but the tree-level scattering amplitudes of the type I superstring restricted to the four-dimensional momenta and polarizations of the gauge bosons. Without performing any explicit calculations, it is obvious that all such amplitudes will have the same threshold scale, namely the mass scale (4.1) of the type I superstring.

Thus, we can summarize our analysis of the $SO(32)$ theory by saying that the apparent heterotic threshold (4.8) is a genuine threshold separating a four-dimensional effective EQFT from a four-dimensional type I superstring theory. No new spacetime dimensions open up at this threshold, but there are infinite towers of massive open and closed string states.

4.3 Very Large Compactifications of the M Theory

Let us now turn our attention to the $E_8 \times E'_8$ theory and confront the biggest puzzle of this paper: *What the devil is a type-I-like threshold scale* (4.8) *doing in the M-theory?* Our answer to this puzzle is that in the $E_8 \times E'_8$ theory, there is no physical threshold at (4.8) and that the $O(R^6/\alpha'_H)$ $F^4_{\mu\nu}$ couplings are artifacts arising from naively integrating out very low mass particles with very weak couplings. This answer is rather surprising from the heterotic point of view — indeed, at the one loop level of the heterotic string there is very little difference between the $E_8 \times E'_8$ and the $SO(32)$ strings and the $F^4_{\mu\nu}$ couplings look virtually identical, — so let us now turn our attention to the dual M-theory.

As explained in section 2, large-radius compactifications of the M-theory have the eleventh dimension compactified on a semicircle of radius $\rho \gg R$ and there is a wide range of intermediate distances ($R \ll L \ll \rho$) at which the world appears to be five-dimensional. In this five-dimensional world, there is $N = 1$ unbroken supersymmetry (presuming the internal sixfold of size R has a Calabi-Yau geometry) and the massless spectrum consists of 1 supergravity multiplet, $\big(h_{11}(CY) - 1\big)$ vector supermultiplet and $\big(h_{12}(CY) + 1\big)$ hypermultiplets.[22−25] When one more dimension is compactified on the semicircle $\mathbf{S^1/Z_2}$, the boundary conditions at $X^{11} = 0$ and $X^{11} = \pi\rho$ differ for different component fields. The components with Neumann conditions at each end of the semicircle comprise the graviton with $d = 4$ indices, one gravitino, $(h_{11} + h_{12} + 1)$ spin $\frac{1}{2}$ fermions and $2(h_{11} + h_{12} + 1)$ real scalars; each of these fields has a massless zero mode as well as an infinite series of massive modes with wave functions $\cos(nX^{11}/\rho)$ and masses n/ρ ($n = 1, 2, 3, \ldots$). The other fields, comprising $(h_{11} + 1)$ four-vectors, $2(h_{12} + 1)$ real scalars, one gravitino and $(h_{11} + h_{12} + 2)$ spin $\frac{1}{2}$ fermions, have Dirichlet boundary conditions at both ends; these fields have massive modes with wave functions $\sin(nX^{11}/\rho)$ and masses n/ρ ($n = 1, 2, 3, \ldots$) but no zero modes.

Altogether, the zero modes of the five-dimensional fields produce the ($d =$

$4, N = 1$) supergravity with a dilaton and $(h_{11} + h_{12})$ moduli superfields but no quarks, leptons, Higgses or any gauge fields of the Standard Model. Instead, all the ordinary particles originate not from the bulk fields of the five-dimensional world but from its boundary at $X^{11} = 0$. The best way to see this is to start with the M-theory in a different regime, namely $\rho \ll \ell_{11} \ll R$ where there is a ten-dimensional effective theory in some intermediate energy range. The gravitational fields of this effective theory originate from the bulk of the eleven-dimensional world, but the $E_8 \times E'_8$ gauge bosons and gaugini originate from the two nine-branes serving as its boundaries. When the $d = 10$ effective theory is compactified to four dimensions, the $d = 10$ gravitational fields and their superpartners give rise to the $d = 4$ SUGRA, dilaton and moduli superfields, but all the ordinary particles come from the $d = 10$ gauge bosons and gaugini of one of the two E_8's; thus the ultimate M-theory origin of the Standard Model is a nine-brane at the boundary of the eleven-dimensional world rather than its bulk.

When we continue the M-theory to the large-radius regime where $\rho \gg R$, the basic picture remains unchanged: The $d = 4$ SUGRA, dilaton and moduli come from the bulk of the $d = 11$ world while the Standard Model* comes from its boundary at $X^{11} = 0$. As to the other boundary at $X^{11} = \pi\rho$, it produces some kind of 'shadow' matter that interacts with the ordinary matter only via gravitational fields propagating through the eleven-dimensional bulk between the two boundaries. When six dimensions $X^5, \ldots, X^{10}$ are compactified on a Calabi-Yau manifold, the eleven-dimensional bulk of the world becomes five-dimensional while the nine-branes at its boundaries become three-branes. *The entire Standard Model lives on one of those three-branes and is oblivious to the bulk of the five-dimensional world or its other boundary.* Likewise, the shadow

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$\star$ Here and henceforth 'the Standard Model' means the ordinary gauge particles, quarks, leptons, Higgses and all their superpartners but not the gravitational or moduli fields. It does not however mean the *Minimal* Supersymmetric SM and may include some non-minimal extensions such as additional $U(1)'$ gauge fields and Higgses that make them massive.

matter lives on the other boundary and is oblivious to both the Standard Model and the five-dimensional bulk; only the gravity and the moduli fields live in five dimensions. Thus, the threshold structure of the $\rho \gg R \gtrsim \ell_{11}$ regime of the M-theory can be summarized as follows:

1. The gravity has a threshold at a rather low energy scale $1/\rho$ (*cf.* eq. (4.2)) above which it becomes five-dimensional. However, this threshold does not affect any gauge, Yukawa or scalar forces of the Standard Model, which remains four-dimensional at distances shorter than ρ and could not care less whether ρ was 10^{-30} cm or 10^{+30} cm or anything in between!

2. The next threshold happens at the Kaluza-Klein scale $M_{\mathrm{KK}} = 1/R$ where six more dimensions open up for both gravity and gauge interactions. Almost immediately above this scale, the effective field theory description breaks down and the fully quantized M-theory (whatever that is) takes over.

Given the above genuine thresholds of the M-theory, one may easily produce a fake threshold at an intermediate scale such as (4.8) by first integrating out the massive modes of the five-dimensional gravitational and moduli fields, then naively extending the resulting $F_{\mu\nu}^4$ operators to energies well above $1/\rho$ until such higher-derivative interactions seem to dominate the scattering amplitudes. Although the couplings of the massive modes are just as weak as those of the ordinary gravity, formally integrating them out produces unexpectedly large $O(\kappa_4^2/\alpha^2\rho^2)F_{\mu\nu}^4$ couplings because the $O(1/\rho)$ masses of those modes are very small. However, the resulting higher-derivative gauge couplings are large only for particle momenta that are smaller than or at most comparable to $1/\rho$; at higher momenta, there are sharply decreasing form factors. If one ignores such form factors, then the $F_{\mu\nu}^4$ couplings appear to dominate the scattering amplitudes at the apparent threshold scale (4.8), but once one takes the form factors into account, this apparent threshold goes away.

A rigorous proof of the above explanation would involve an all-order calcu-

lation of all the $F^A_{\mu\nu}$ couplings and their form factors in both heterotic $E_8 \times E_8'$ string theory and M-theory and verifying that they agree with each other. Such an all-order calculation is beyond our technical abilities, so we shall limit our evidence to verifying that the tree-level M-theory yields the same zero-momentum $F^4_{\mu\nu}$ couplings in $d = 4$ as the one-loop-level heterotic string. In the M-theory picture, we expect the $F^4_{\mu\nu}$ couplings to arise at the $d = 4/d = 5$ threshold, so let us consider how the bulk five-dimensional fields interact with the gauge fields living on the four-dimensional boundaries.

From the five-dimensional point of view, the effective action for both bulk and boundary fields must have general form

$$\int d^5x\,\mathcal{L}_5\,[\text{bulk fields}] \;+\; \int d^4x\,\mathcal{L}_4\,\big[\text{boundary and bulk fields at } X^{11} = 0\big]$$

$$+\; \int d^4x\,\mathcal{L}_4'\,\big[\text{boundary and bulk fields at } X^{11} = \pi\rho\big]\,.$$

$$(4.12)$$

Note that each bulk field component has either Dirichlet boundary conditions on both boundaries or Neumann conditions on both boundaries. According to eq. (4.12), the components with the DD boundary conditions do not couple to any boundary fields, so we may safely drop them from our considerations. All the remaining components thus have NN boundary conditions and hence zero modes in four dimensions, and furthermore, all the massive modes of any bulk component couple to the boundary fields exactly like the corresponding zero mode, *i.e.*, through combinations of the form

$$\Psi(X^{11} = 0) \;=\; \sum_{n=0}^{\infty} \Psi_n \qquad \text{and} \quad \Psi(X^{11} = \pi\rho) \;=\; \sum_{n=0}^{\infty} (-1)^n \Psi_n\,. \qquad (4.13)$$

Consequently, all the interactions between the boundary fields and the bulk five-dimensional fields can be read from the low-energy four-dimensional effective Lagrangian (in which the bulk $d = 5$ fields are represented via their zero modes) without any additional input from the $d = 5$ effective theory or the M-theory itself.

At the tree level (of the heterotic string and of the low-energy EQFT), the four-dimensional gauge fields couple to the graviton, the dilaton and the axion, but do not couple to the moduli of the Calabi-Yau sixfold. Consequently, at the linearized level, their couplings to the canonically-normalized massive modes of the corresponding bulk fields can be summarized as

$$\frac{\sqrt{2}\kappa_4}{16\pi\alpha_{\text{GUT}}} \sum_{n=1}^{\infty} \sum_{a} k_a (\pm 1)^n \left[G_n^{\mu\nu} \left(2\,\text{tr}(F_{\mu\lambda} F_\nu^{\ \lambda}) - \tfrac{1}{2} g_{\mu\nu}\,\text{tr}(F_{\kappa\lambda} F^{\kappa\lambda}) \right) \right.$$
$$\left. + D_n\,\text{tr}(F_{\mu\nu} F^{\mu\nu}) + A_n\,\text{tr}(F_{\mu\nu} \tilde{F}^{\mu\nu}) \right]_a . \tag{4.14}$$

When those massive modes are integrated out, the couplings (4.14) result in the $F_{\mu\nu}^4$ interactions in the four-dimensional effective Lagrangian; for the four-momenta much smaller than the n/ρ masses of the massive modes, we have

$$\mathcal{L}_{F_{\mu\nu}^4} = -\left(\frac{\kappa_4}{16\pi\alpha_{\text{GUT}}} \right)^2 \sum_{a,b} N_{a,b} \left[4\,\text{tr}(F_{\mu\lambda} F^{\nu\lambda})_a\,\text{tr}(F^{\mu\kappa} F_{\nu\kappa})_b \right.$$
$$\left. + \text{tr}(F_{\mu\nu} \tilde{F}^{\mu\nu})_a\,\text{tr}(F_{\kappa\lambda} \tilde{F}^{\kappa\lambda})_b \right] \tag{4.15}$$

where

$$N_{a,b} = k_a k_b \sum_{n=1}^{\infty} \frac{(\pm 1)^n \rho^2}{n^2} = \begin{cases} \frac{\pi^2}{6} \rho^2 k_a k_b & \text{for } a \text{ and } b \text{ in the same } E_8, \\ -\frac{\pi^2}{12} \rho^2 k_a k_b & \text{for } a \text{ and } b \text{ in different } E_8\text{'s.} \end{cases} \tag{4.16}$$

Identifying each $d = 4$ gauge group's factor G_a as a level k_a subgroup of either E_8 or E_8' and performing some straightforward (if tedious) manipulations of the gauge and space-time indices, we re-write eq. (4.15) as

$$\mathcal{L}_{F_{\mu\nu}^4} = -\mathcal{N}\tau^{1234} \left(\text{tr}(F_1 F_2)\,\text{tr}(F_3 F_4) - \text{tr}(F_1 F_2)\,\text{tr}(F_3' F_4') + \text{tr}(F_1' F_2')\,\text{tr}(F_3' F_4') \right) \tag{4.17}$$

where τ^{1234} is exactly as in eq. (4.4) (except for the restriction to the four-

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dimensional indices) and the overall coefficient is

$$\mathcal{N} = \frac{\pi^2 \rho^2}{12} \left(\frac{\kappa_4}{16\pi\alpha_{\mathrm{GUT}}} \right)^2 = \frac{\pi}{768} \frac{R^6 \rho}{\ell_{11}^3} = \frac{\pi}{384} \frac{R^6}{\alpha'_H} \tag{4.18}$$

(*cf.* eqs. (3.4) and (3.2)), in exact agreement with the heterotic one-loop formula (4.3) $\times (2\pi R)^6$.

In the M-theory picture, it is quite obvious that the low-energy couplings (4.17) have rapidly decreasing form-factors for the four-momenta larger than $1/\rho$, but this behavior is anything but obvious in the dual heterotic picture. Indeed, while from the M-theory point of view, the form factor is some analytic function of $t\rho^2$ (t being the four-momentum-square of the gauge-singlet channel),[★] from the heterotic point of view, the same form factor becomes a function of $2t\alpha'_H \hat{\lambda}_H^2$. Thus, it cannot be obtained at any finite heterotic loop order but only from summing the entire perturbative theory and seeing that the series not only converges but in fact decreases with t. Needless to say, we have not performed such an all-loop calculation; however, having reproduced the heterotic one-loop result as the zero-momentum limit of the M-theory, we have enough confidence in the heterotic $\leftrightarrow$ M-theory duality to conclude that the apparent threshold at the (4.8) scale is indeed an artifact of the one-loop approximation.

The real puzzle about the one-loop heterotic prediction for the threshold at (4.8) is not so much why it fails in the $E_8 \times E'_8$ case but rather why it fails in the

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[★] Specifically, the $F_{\mu\nu}^4$ form factor is

$$\frac{3}{\pi^2 \rho^2 (-t)} \left[\frac{\pi\rho\sqrt{-t}}{\tanh \pi\rho\sqrt{-t}} - 1 \right] \approx \frac{3}{\pi\rho\sqrt{-t}} \quad \text{for } \rho^2 |t| \gg 1$$

for the four gauge bosons belonging to the same E_8 and thus originating from the same $d = 4$ boundary of the five-dimensional spacetime. For the gauge bosons originating on two different boundaries and hence belonging to different E_8's, the $F_{\mu\nu}^4$ form factor is

$$\frac{6}{\pi^2 \rho^2 (-t)} \left[1 - \frac{\pi\rho\sqrt{-t}}{\sinh \pi\rho\sqrt{-t}} \right] \approx \frac{6}{\pi^2 \rho^2 (-t)} \quad \text{for } \rho^2 |t| \gg 1.$$

$E_8 \times E_8'$ case and yet holds true in the $SO(32)$ case. From the conformal theory point of view, the two heterotic string theories are Z_2 orbifolds of each other in ten dimensions and their toroidal compactifications are T-dual to each other. Consequently, any orbifold compactification of the $SO(32)$ heterotic string can also be constructed as an orbifold of the $E_8 \times E_8'$ string and vice verse. In light of this interrelatedness between the two heterotic strings, the only explanation we can offer for their very different threshold structures in the large-radius limit is that perhaps the large R limits of the two strings are not equivalent but rather T-dual to each other (or T-dual for some of the six internal coordinates but equivalent for the others). Consequently, the properties of the two strings that appear similar at the lowest non-trivial loop order (one loop for the $F_{\mu\nu}^4$ couplings) may behave quite differently at the higher loop orders. Verifying this conjecture is however beyond the scope of the present article.

4.4 Phenomenological Limits on the Compactification Size

In this last section of the paper, we are finally ready to answer the big question: *What is the largest possible size of the internal sixfold in a realistic compactification?* We have already seen that the size of a generic compactification of either $SO(32)$ or $E_8 \times E_8'$ heterotic string is limited on theoretical grounds by eq. (2.13). However, in both heterotic strings this limitation has loopholes allowing some non-generic internal sixfolds to grow arbitrary large. What then are the phenomenological limits on their sizes?

In the $SO(32)$ case, the key to the phenomenological limits is the type I superstring threshold scale (4.1). Experimentally, there is no such threshold at any energies explored by the present-day accelerators. Furthermore, high-precision tests of the Standard Model rule out stringy form factors corresponding to $\alpha' \gtrsim 1 \text{ TeV}^{-2}$. According to eq. (2.5), this consideration alone is sufficient to require $R < 3 \cdot 10^{-22}$ cm, *i.e.*, $M_{\text{KK}} > 7 \cdot 10^7$ GeV.

Our next concern is with the baryon stability. Baryon number conservation cannot be an exact symmetry of the type I superstring, even at the tree (disk) level, since all known ways of enforcing such a conservation law result in a gauged rather than global $U(1)$ symmetry.[†] Generically, a B-violating operator of canonical dimension D should have an $O(\ell_I^{D-4})$ coupling, but for many string models, some of the possible B-violating operators would be absent because of a custodial discrete symmetry or because of a variety of stringy reasons; consequently, the bound on the superstring threshold scale imposed by the observed baryon stability is highly model dependent. Consider a few examples:

- A $D = 5$ supersymmetric F-term $M_B^{-1}[QQQL]_F$ would result in unacceptably high baryon decay rate for any $M_B < M_{\text{Planck}}$. Such $D = 5$ B-violating operators must be avoided in any realistic string model, regardless of the internal manifold's size.

◇ Many models without the $D = 5$ B-violating operators have $D = 6$ four-fermion operators such as $M_B^{-2} qq\bar{u}\bar{e}$ produced directly at the string scale (without any "supersymmetric dressing"). Experimental limits on such operators[26] are $M_B^2 \gtrsim 10^{31}$ GeV^2, which in the present context would imply $\alpha_I' < 10^{-31}$ GeV^{-2} and $M_{\text{KK}} \gtrsim 10^{16}$ GeV.

⋆ Now consider a model where B is conserved modulo 2 or where $|\Delta B| = 1$ operators are absent for some other reason. In this case, the leading B-violating operator would be a $D = 7$ F-term such as $M_B^{-3}[UDDUDD]_F$, which after suitable "supersymmetric dressing" would cause neutron $\leftrightarrow$ antineutron oscillations as well as lepton-less double baryon decay in nuclei. Phenomenologically,[27,28] $G[n \leftrightarrow \bar{n}] < 10^{-27}$GeV^{-5}, which for the $O(100$ GeV) squark and gluino masses implies $M_B \gtrsim 10^6$ GeV. For our

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† In principle, there could be a 'fifth force' due to a gauged $U(1)_{\text{Baryon}}$, but the coupling of such a force must be much weaker than the couplings of baryons to gravity, $\alpha_B \ll (M_B/M_{\text{Planck}})^2 \sim 10^{-38}$. By comparison, the weakest gauge coupling one may expect to find in a large-radius compactification of the $SO(32)$ heterotic / type I theory is $\alpha_{\text{min}} = O(\ell_H^2/R^2)$ (cf. eq. (2.16)) $\gtrsim 10^{-20}$, which is not weak enough for the fifth force.

purposes, this means that the type I superstring threshold could be as low as a million GeV and the Kaluza-Klein scale $1/R$ could be as low as 10^{10} GeV.

Presumably, there exist string models with even more restricted B-violating operators. Such models would tolerate even lower superstring thresholds, and perhaps even a TeV-ish threshold would be allowed in a few models.

The neutrino masses are also sensitive to the very-high-energy physics via the see-saw mechanism, which gives

$$m_\nu \sim \frac{m_{\mathrm{ew}}^2}{m_{\mathrm{high}}} \tag{4.19}$$

where m_{ew} is some kind of an electroweak mass. Unfortunately, in the absence of a string-theoretical explanation of the mass hierarchy between the three generations of quarks and charged leptons, it is not clear whether m_{ew} in eq. (4.19) is similar to M_W or to the mass of the charged lepton of the appropriate generation. In the former case, the present-day experimental limits on neutrino masses would require $m_{\mathrm{high}} > 10^{12}$ GeV, while in the latter case the neutrinos would be light enough for any m_{high} above the weak scale. It is also possible to have $m_{\mathrm{ew}} = 0$, in which case, the neutrinos are exactly massless regardless of the m_{high}. Therefore, while the neutrino masses may constrain the threshold scales in some string models, they do not impose any general, model-independent constraints beyond $M_I > O(1 \text{ TeV})$. Likewise, the experimental limits on various flavor-changing neutral currents may rule out some string models with TeV-ish thresholds, but the string-theoretical couplings of such currents are so model-dependent that no general conclusion is possible.

Finally, let us consider the running of the three gauge couplings of the Standard Model, which appear to unify (at levels $k_3 = k_2 = 1$, $k_1 = 5/3$) at $M_{\mathrm{GUT}} \sim 10^{16}$ GeV. Again, the implication of this apparent trinification are too model-dependent to impose a general constraint on the thresholds of the compactified $SO(32)$ heterotic / type I theory: Indeed, even if we assume that there

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are no field-theoretical intermediate-scale thresholds between the weak scale and the type I superstring scale, it still does not follow that the three Standard gauge couplings unify at the type I threshold. Instead, we may have $O\big(\log(\alpha'_I/R^2)\big)$ threshold corrections that just happen to be proportional to the four-dimensional β-functions of the couplings. Consequently, the three couplings would appear to unify at the scale

$$M_{\rm GUT}^{\rm fake} \ \sim \ M_I \times (RM_I)^{\rm some\ power} \tag{4.20}$$

— which may or may not have any physical meaning — even though the actual threshold is at $M_I = 1/\ell_I$ rather than at this apparent GUT scale. Notice that such a fake grand unification is far from uncommon in string theory: For example, in some orbifold compactifications of the heterotic string, the four-dimensional gauge couplings appear to unify at $M_{\rm GUT} \sim 1/\ell_H$ even though the four-dimensional EQFT breaks down at the Kaluza-Klein scale $1/R$. Unfortunately, we do not know enough about $R \ll \ell_I$ compactifications of the type I superstring to give specific examples of fake grand unification in such string models, or even to tell whether the apparent GUT scale is likely to coincide with the heterotic string scale $1/\ell_H$ or perhaps with the Kaluza-Klein scale $1/R$. However, barring unexpected cancellations, we do expect to have $O\big(\log(\alpha'_I/R^2)\big)$ threshold corrections to the running gauge couplings[*] and hence any apparent grand unification does not necessarily pose a constraint on the physical threshold scales of the string theory.

Now consider the $E_8 \times E_8'$ theory, which in the large R limit has two distinct thresholds — the $(d = 4) \to (d = 5)$ threshold at $1/\rho$ and the $(d = 5) \to$

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[*] According to eq. (2.16), the Wilsonian gauge couplings either go haywire in the large R limit — which we assume not to happen — or else have only $O(1)$ string threshold corrections. However, the physical running gauge couplings have additional non-holomorphic threshold corrections which depend on the the Kähler function of the low-energy EQFT. Generally, if there are light charged particles whose wave function normalizations are proportional to powers of the radius R and/or if the Kähler function for the moduli fields has a $\log R^2$ term, then the non-holomorphic threshold corrections would grow like $\log(R^2/\alpha'_H)$ in heterotic terms — or like $\log(\alpha'_I/R^2)$ in the type I terms.

$(d = 11) \to$ M-theory threshold at $(1/R) \sim (1/\ell_{11})$ — but the Standard Model is oblivious to the first threshold and continues to live on a three-brane all the way to the second threshold. However, the first threshold is quite physical as it changes the behavior of the gravitational force; this change is not limited to relativistic gravity but would be quite apparent in any static Cavendish-like experiment at distances comparable to or than smaller than, the five-dimensional width $\pi\rho$. Specifically, instead of the Newtonian force, one has

$$f_{12} = \frac{G_N m_1 m_2}{r^2} \times \frac{1 + \left(\frac{r}{\rho} - 1\right) e^{-r/\rho}}{\left(1 - e^{-r/\rho}\right)^2} \tag{4.21}$$

where the short-distance corrections are due to the massive modes of the graviton. Comparing this expression with the experimental upper limits on Yukawa-like 'fifth forces',[29] we find $\rho < 2$ mm.

Remarkably, this almost human-scale limit on the fifth dimension of the M-theory is sufficient to put the other six compact dimensions quite out of reach of any presently contemplated accelerator: According to eq. (3.4), $\rho < 2$ mm translates into $R < 5 \cdot 10^{-22}$ cm or $M_{KK} > 4 \cdot 10^7$ GeV! In fact, this limit is stronger than any general, model-independent limit obtained from the non-gravitational Standard Model phenomenology, although many particular types of string models are subject to much more stringent limitations.

For example, in *smooth* Calabi-Yau compactifications of the heterotic string, the one-loop running gauge couplings appear to unify at $M_{\text{GUT}} \sim M_{KK}$[11] (assuming $C_a = 0 \ \forall a$ since otherwise the compactification could not be very large) and it is difficult to imagine any other unification scale emerging from the dual M-theory whose only *relevant* threshold is at M_{KK}. If such a smooth compactification also has the conventional embedding of the low-energy $SU(3) \times SU(2) \times U(1)$ into the E_8 and no intermediate-energy Higgs-like thresholds, then such a model must have $M_{KK} \sim 10^{16}$ GeV. However, there are many ways a model could avoid this limitation: There may be an intermediate-energy threshold, or the

gauge coupling may unify in a non-conventional way (*e.g.*, with $k_2 = 2$) because of a non-minimal embedding of the $SU(3) \times SU(2) \times U(1)$ into the E_8, or the compactification may be non-smooth. In the latter case, without going into the (not yet understood) details of the singular compactifications of the M-theory, it stands to reason that a charged particle arising from a fixed point of an orbifold or a similar singularity would have a very different normalization of its wave function than a particle arising from a bulk mode of the internal sixfold. The largish logarithm of this normalization would then result in largish non-holomorphic threshold corrections to the running gauge couplings, which would in turn shift the apparent GUT scale from $O(1/R)$ to something entirely different, perhaps to the dual heterotic string scale $M_H = 1/\ell_H$. Indeed, the heterotic calculations suggest $M_{\mathrm{GUT}} \sim M_H$ for any orbifold with $C_a = 0 \;\forall a$, although the validity of this result in the $\hat{\lambda}_H \gg 1$ regime is yet to be established.

All other experimental limitations on the compactification scale of the $E_8 \times E_8'$ theory are completely analogous to the limitations on the type I superstring scale in the $SO(32)$ case. Given the gravitational limit $M_{\mathrm{KK}} > 4 \cdot 10^7$ GeV, the flavor-changing neutral currents are certain to be well below the experimental limits while the neutrino masses may be problematic for some models with non-hierarchical $m_{\mathrm{ew}} \gtrsim 1$ GeV. The baryon-number-violating operators with $|\Delta B| \geq 2$ (and hence $D \geq 7$) are also safe, but any $D \leq 6$ B-violating operator would render a large-radius compactification quite unrealistic. Again, this is a powerful phenomenological constraint on specific large-radius compactifications, but it can be easily satisfied by any string model with an exact $(-1)^B$ symmetry or some other custodial symmetry prohibiting $|\Delta B| = 1$ processes. Altogether, it is quite possible for the internal sixfold to be as large as $5 \cdot 10^{-22}$ cm, although there is no phenomenological reason to prefer so large a size.

4.5 Caveats and Speculations in Five Dimensions

After a dozen or so years of modern string theory, there are still no working solutions to the twin problems of hierarchical supersymmetry breaking in four dimensions and of stabilizing the vacuum values of the dilaton and the compactification moduli although several general scenarios have been proposed. The more popular scenarios rely on non-perturbative effects produced by confining 'hidden' gauge forces and other purely field-theoretical four-dimensional phenomena. Alternatively, it is possible that the $d = 4$ SUSY is broken and the vacuum degeneracy is lifted by some inherently stringy (or M-theoretical, *etc.*) non-perturbative effects which happen to be hierarchically weak because they involve some kind of a large-action instanton. Currently, new kinds of stringy non-perturbative effects are discovered and analyzed weekly if not daily, so we expect the non-EQFT scenarios for SUSY breaking to receive more attention in the near future. At the moment however, implications of the large internal dimensions for such scenarios are far from clear.

Let us therefore focus on the scenarios where a confining hidden gauge force (or several such forces) either breaks $d = 4$ supersymmetry dynamically or else generates a dynamical superpotential for the moduli superfields (including the dilaton/axion S) that leads to a spontaneous SUSY breakdown in the moduli sector. In all such scenarios one implicitly assumes that Λ_{hid} — the confining scale of the hidden forces — is well below any string or Kaluza-Klein threshold.[*] For the large-size compactifications of the $SO(32)$ heterotic $\leftrightarrow$ type I theory, this means $\Lambda_{\text{hid}} \ll M_I$. In particular, the Dine-Nelson scenario[30] where SUSY is dynamically broken at few tens of TeV requires $M_I \gg 100$ TeV and hence $M_{\text{KK}} \gg 10^9$ GeV; the limits are higher in other scenarios, which involve hidden forces with higher confining scales.

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[*] Actually, the supersymmetry breaking effects may well continue without a phase transition into the regime of $\Lambda_{\text{hid}} \gtrsim M_{\text{string}}$. Unfortunately, the state-of-the-art techniques for analyzing dynamical SUSY breaking do not work in that regime.

In the $E_8 \times E_8'$ theory, the confining forces live on a three-brane boundary of the five-dimensional space — or possibly on both boundaries — and are oblivious to the $(d = 4) \to (d = 5)$ threshold at ρ^{-1}, so naively, the only limitation on the compactification size is $M_{\text{KK}} \gg \Lambda_{\text{hid}}$. However, for $\rho \gtrsim \Lambda_{\text{hid}}^{-1}$, one faces a whole host of questions about the moduli and gravitational fields which live in the five-dimensional bulk and act five-dimensional at energies $O(\Lambda_{\text{hid}})$:

- If SUSY is dynamically broken on the three-brane boundary, how does the resulting energy density affect the $d = 5$ supergravity between the branes?

⋈ If the dynamical SUSY breaking happens on the 'shadow' E_8' boundary, how do the moduli and the gravitational fields communicate this break-down to the Standard Model living on the other boundary?

⋆ If the confining hidden force does not break SUSY by itself but generates a superpotential for the moduli, how does this affect the massive modes of the moduli? Or, from the five-dimensional point of view, what kind of moduli-field *gradients* does one get in this scenario?

⋄⋄ How does this five-dimensional mess communicate SUSY breakdown to the Standard Model?

⋆⋆ What happens if there are confining hidden forces on both three-brane boundaries of the $d = 5$ space?

∞ Given that the non-perturbative effects happen on the three-brane boundaries, what mechanism stabilizes the fifth dimension's width $\pi\rho$? In particular, what if anything prevents the five-dimensional runaway $\rho \to \infty$?

— And on top of all these questions about the equilibrium state of the five-dimensional Universe, one should also consider its cosmological history.

A pessimist pondering the above questions would conclude that realistic $\rho \gtrsim \Lambda_{\text{hid}}^{-1}$ compactifications of the M-theory are improbable. An optimist looking at the same questions would see novel scenarios for supersymmetry breaking that might end up working better than any purely four-dimensional scenarios (which

do not work all that well). The present authors see in these questions a subject for future research.

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